On Lattice Point Weak *b*-Visibility

Part 3/3: George Kuliner and Patrick Crossley

What we will be discussing:

How to find a point that weakly b-views a lattice that meets certain criteria

An example of using the process

Finding the closest point that weakly b-views the restricted lattice

Where our research leads to

Theorem 2.7. *Fix $b \in \mathbb{N}$ and let $m, n \in \mathbb{N}$ satisfy $m - 1 > n + 1$. Then the set $\Delta_{m,n}$ is weakly b-visible from the point $P = (m(m-1), n+1)$.*

Example: Consider *b=2* and the delta region, $\Delta_{6,3}$. Can we find a point P such that every point in the delta region weakly 2-views point P?

Let's Test Our Theorem Out:

Yes. Notice m=6, n=3 satisfying 6-1>3+1. Then using Theorem 2.7, the point P=(6(6-1),3+1)=(30,4) should be weakly 2-view the delta region.

(29,2)	(29,1)
(7*2^2,2)	(7*2^2,1)
(3^3,2)	(3^3,1)
(13*2,2)	(13*2,1)
(5^2,,2)	(5^2,1)
(3*2^3,2)	(3*2^3,1)

Can We Get Closer?

Corollary 2.7.1. *Fix $m-1 > n+1$ and $b \in \mathbb{N}$ and let $f(x) = \frac{(n-1)}{(m-1)^b}(x-1)^b + 1$. If $m^* > m$ is the smallest positive integer such that $(n, f(m^*)) \cap \mathbb{N} \neq \emptyset$, then $\Delta_{m,n}$ is weakly b-visible from $P = (m^*, n+1)$.*

Proof. If $P = (m^*, n+1)$, where $m^* > m$ is the smallest positive integer such that $(n, f(m^*)) \cap \mathbb{N} \neq \emptyset$, then P satisfies the conditions of Theorem 2.7. Thus P weakly b-views $\Delta_{m,n}$. Note that the coordinates of P will be minimal among any points lying below L and H (as in the proof of Theorem 2.7). Thus the distance from $P = (m^*, n+1)$ to $\Delta_{m,n}$ will be minimal. $\square$

What Questions did our Research Lead To?

Can we find a closest point to any type of region with no restrictions of m and n?

Can we find points of visibility with other types of functions: sinusoidal, polynomial, rational, logarithmic?

Given a point P what are all the regions in which the point weakly views?

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Questions?